Whitening

Introduction

They say that whitening composes a half of ICA(Independent Component Analysis)'s process[1]. For the author, he has not ever experienced that ICA works well. As the reason, the author often encounters that the solution varies according to the initial values, or the solution appears by multiple. However, the data type might not be suitable for ICA, or the self made program(C++) might have bugs.

Aside from that, as the preprocessing of ICA, PCA(Principal Component Analysis) and the whitening will be applied generally. If the professional ICA was not used, this preprocessing would have the strong points.

＊Solution stability

＊Solution uniqueness

This paper describes the whitening availabilities.

PCA(Principal Component Analysis)

The paper describes the principal component analysis simply. The principal component analysis is only an eigen value problem in a nutshell. In detail, see the sequence below.

① Calculate the variances, covariances of the gotten data, then construct the covariance matrix C

It is needless to say the covariance's calculation formula, this paper omits it. In addition, for the author's case, the programming statements are unified by unbiased covariance(the other hand is population variance).

② Solve the eigen values' vector L for the covariance matrix C(1)

$$L = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \end{bmatrix} \quad (1)$$

n: Dimension(number of data series)

In the numerical calculation, Hessenberg matrix will be calculated by Householder transform, then QR decomposition will be applied to this. For the covariances which are symmetric, Householder transform for the symmetric version is sufficient(There is an asymmetric version too). There is Jacobi method's equipment, but the author does not use Jacobi method. The paper omits the detail algorithm.

③ Solve the eigen vectors' matrix E from the eigen values. The solution is by (2)

$$CV_i = \lambda_i I \quad (2)$$

$$E = [V_1 \quad V_2 \quad \cdots \quad V_n]$$

$$V_i = \begin{bmatrix} v_{i1} \\ v_{i2} \\ \vdots \\ v_{in} \end{bmatrix}$$

The numerical solution might be Gaussian elimination.

At last, the covariance matrix will be expressed by (3).

$$C = EDE^{\top} \quad (3)$$

$$D = \mathrm{diag}(L) = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \lambda_n \end{bmatrix}$$

Now, the eigen value becomes each variance of data series, so this is a part of the functional beauty which appears in PCA. In addition, the unitary matrix has (4) as a characteristic.

$$(UDU^{\top})^{-1} = UD^{-1}U^{\top} \quad (4)$$

Thanks to this characteristic, an inverse matrix can be solved from the eigen value decomposition. This may be off the track just a bit, solving the inverse matrix is not very welcome in the numerical calculation. However, it is often used in the verification.
For the author, Gauss Jordan is used frequently. This is the sequence like an extension version of the gaussian elimination. Although the simple algorithm, the pivoting is required as same as the gaussian elimination.

Whitening

Whitening matrix W is defined by (5).

$$W = C^{-\frac{1}{2}} = (EDE^{\top})^{-\frac{1}{2}} = ED^{-\frac{1}{2}}E^{\top} \quad (5)$$

$$D^{-\frac{1}{2}} = \begin{bmatrix} \frac{1}{\sqrt{\lambda_1}} & 0 & \cdots & 0 \\ 0 & \frac{1}{\sqrt{\lambda_2}} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \frac{1}{\sqrt{\lambda_n}} \end{bmatrix}$$

The squared part of diagonal components means the normalization which makes the variance value 1. Here, the whitening is to multiply the whitening matrix to the observation data(later mention).

Cocktail Party Problem

The cocktail party problem is exactly the famous subject for that Independent Component Analysis(**Fig.1**).

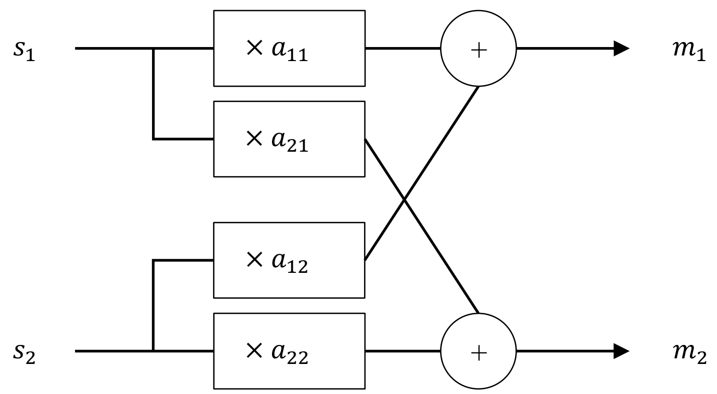

Fig.1 Cocktail Party Problem

Assume the audio signals s_1, s_2, then the composition is that 2 microphone m_1, m_2 receive them. This is expressed by the matrix (6).

$$AS = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} = M \quad (6)$$

As the problem, it is to estimate A and S from M. Although 2 variables at (6), it may be any multivariable. This subject will be handled in the simulation. For the whitening, translate the observation data by (7).

$$WM \quad (7)$$

Data will be decorrelated by the whitening. The decorrelation is to make the covariance value 0(zero). In the other hand, all the diagonal components becomes 1 by normalization(5).

Whitening Simulation

For the author, it is restricted that he does not have any effective data for ICA. Let's verify the effect of whitening by the appropriate periodic waveforms(8)～(10).

Condition:

$$s_1 = \cos\left(2\pi \frac{i}{125}\right) + 2 \quad (8)$$

$$s_2 = 0.01(i\,\%100) + 2 \quad (9)$$

$$i = 0, 1, 2, \cdots, 499$$

$$A = \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.9 \end{bmatrix} \quad (10)$$

Each signal is shown in **Fig.2～Fig.5**.

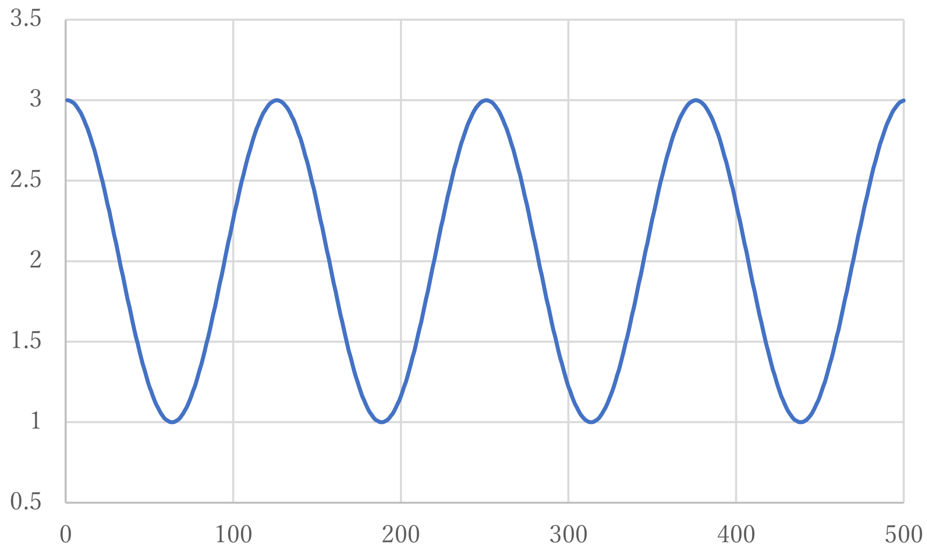

Fig.2 s_1

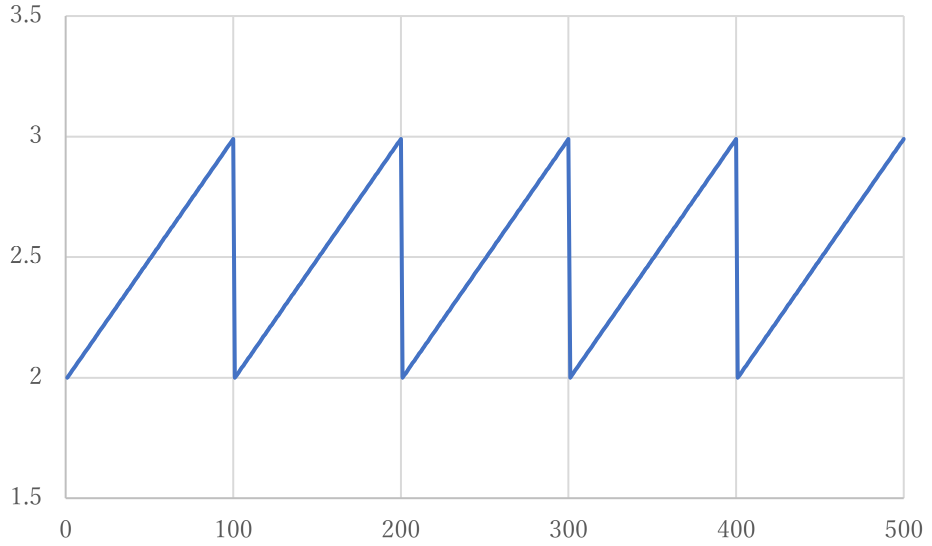

Fig.3 s_2

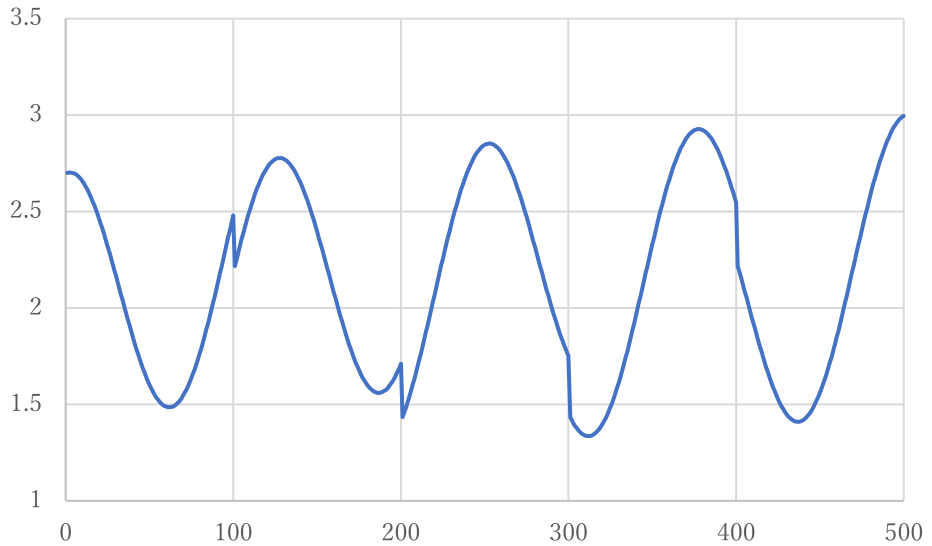

Fig.4 m_1

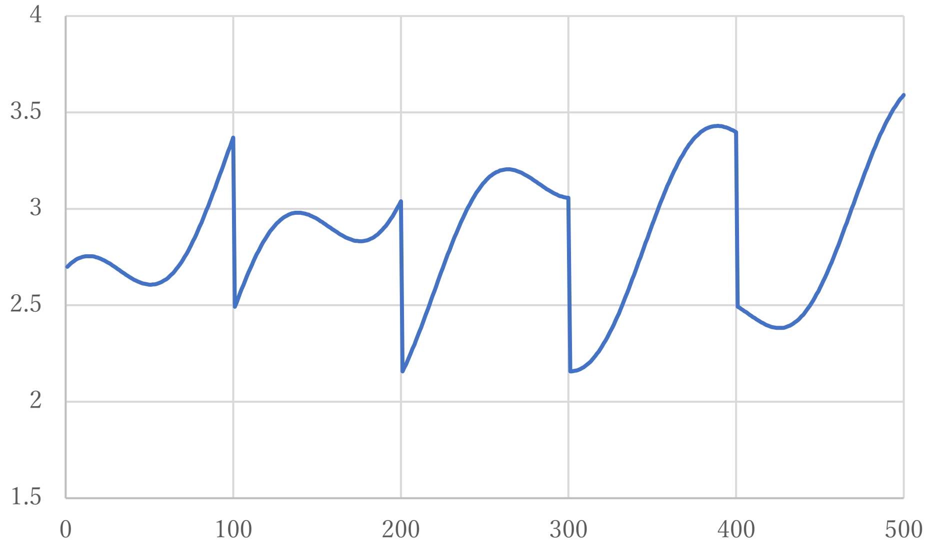

Fig.5 $\boldsymbol{m_2}$

Simulation Result

The whitening result is shown in **Fig.6**, **Fig.7**.

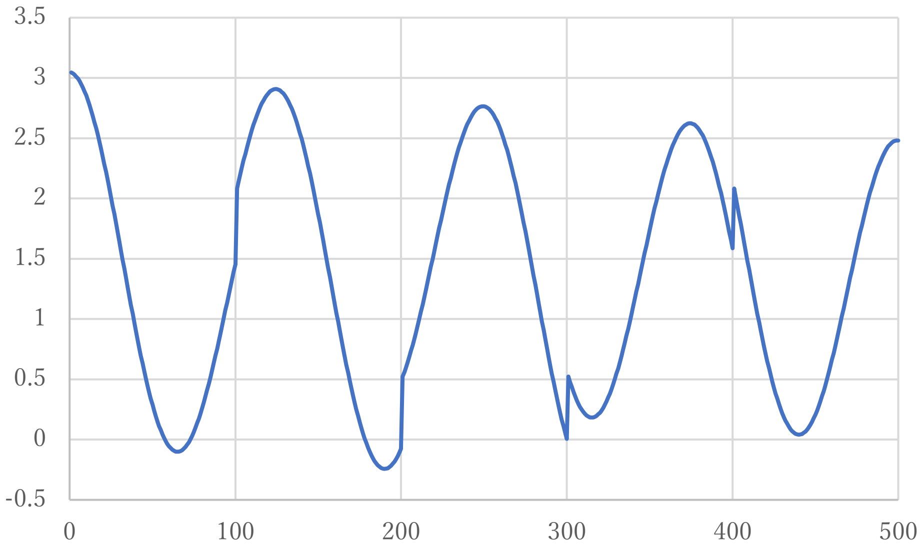

Fig.6 $\boldsymbol{s_1}$ **Estimation by Whitening**

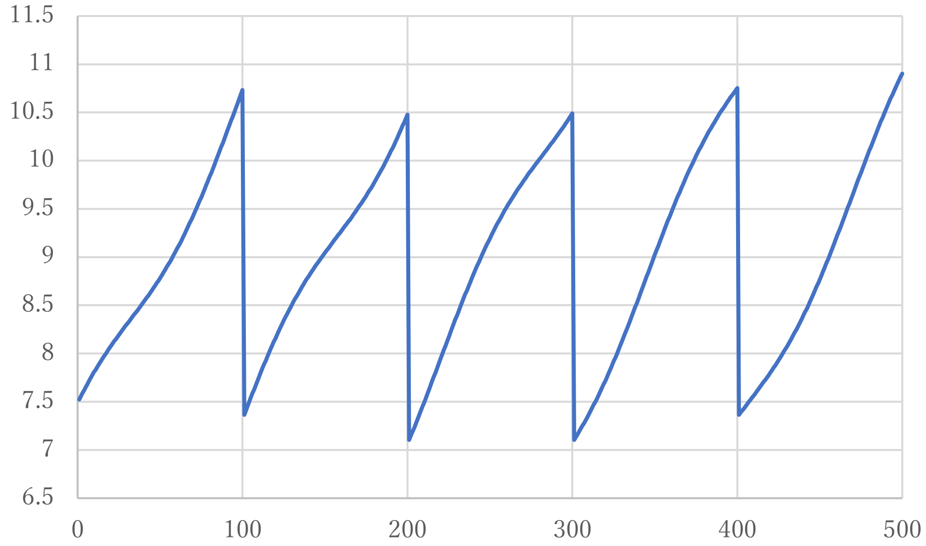

Fig.7 s_2 Estimation by Whitening

Originally in (6), A and S are unknown, so their absolute values can not be known. Therefore, ignore the amplitude intensity, please pay attention to the signal shape. By the whitening, it is estimated similar to the source signal shape, even if not completely.

3 Signals Simulation

Assume 3 signals by adding one more signal, simulate again(11), (12), **Fig.8～Fig.11**.

Condition:

$$s_3 = (i \,\%25 > 12)?\,2:1 \quad (11)$$

$$A = \begin{bmatrix} 0.7 & 0.3 & 0.5 \\ 0.2 & 0.5 & 0.8 \\ 0.1 & 0.7 & 0.3 \end{bmatrix} \quad (12)$$

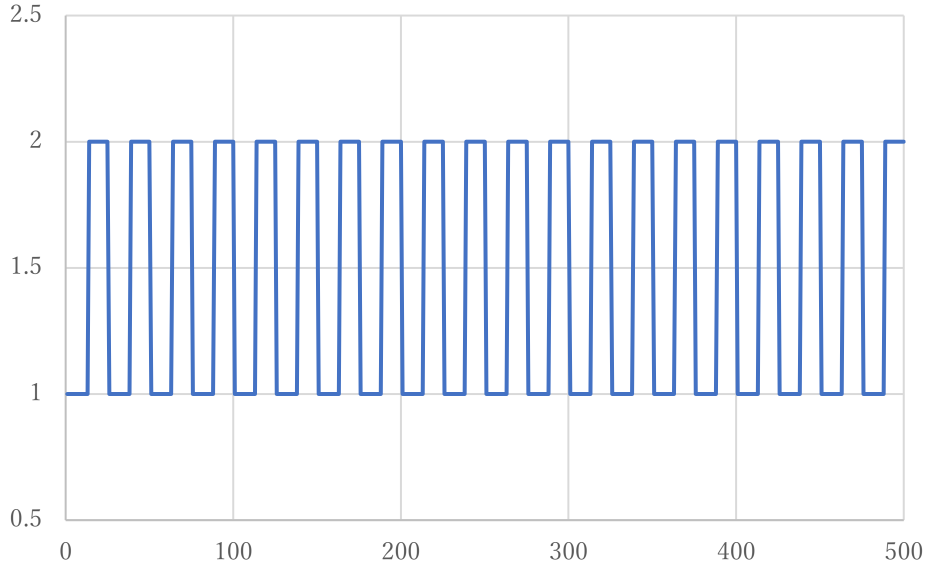

Fig.8 s_3

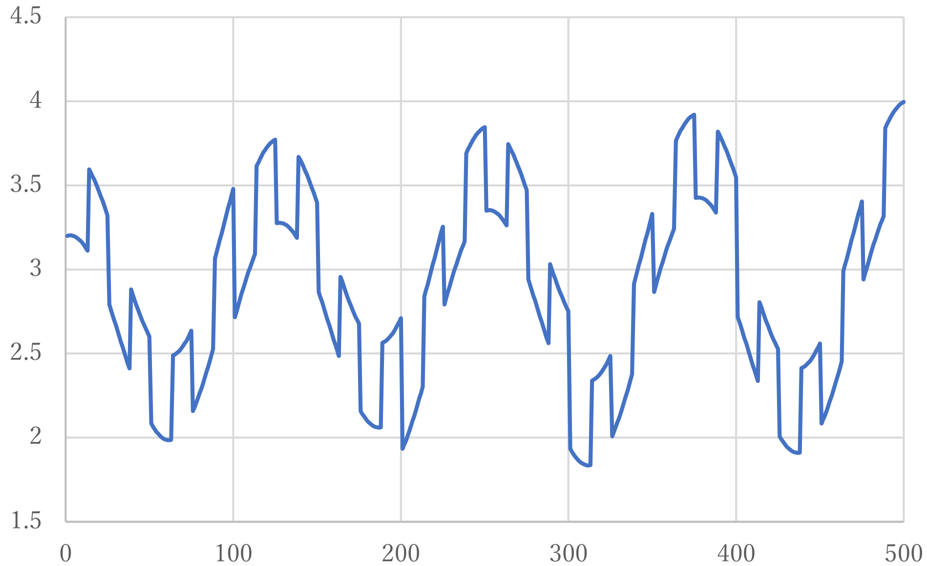

Fig.9 m_1

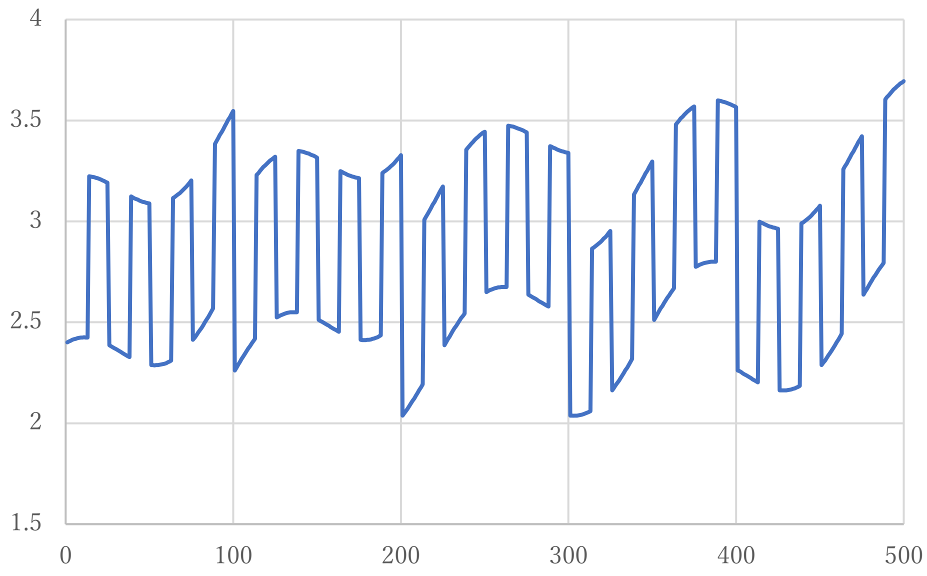

Fig.10 $\boldsymbol{m_2}$

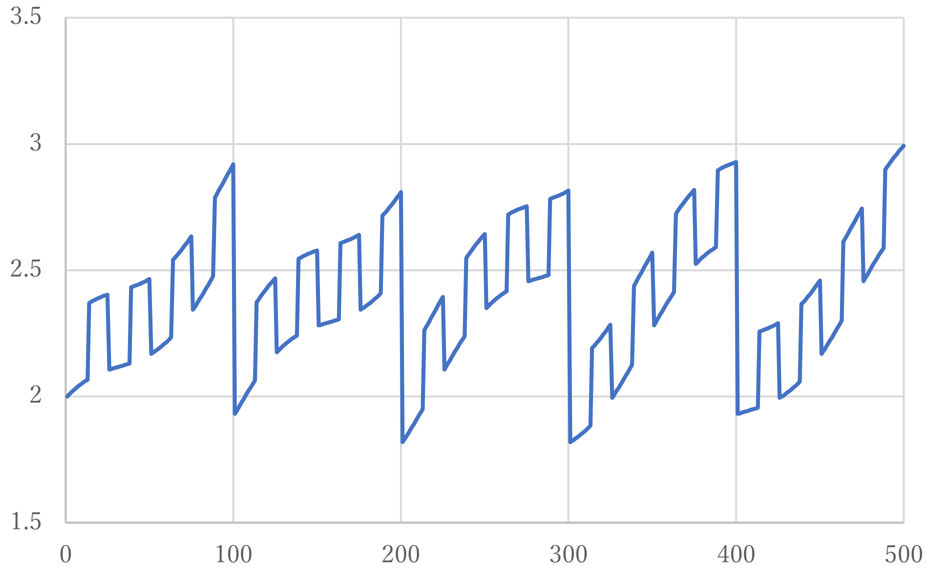

Fig.11 $\boldsymbol{m_3}$

Simulation Result for 3 Signals

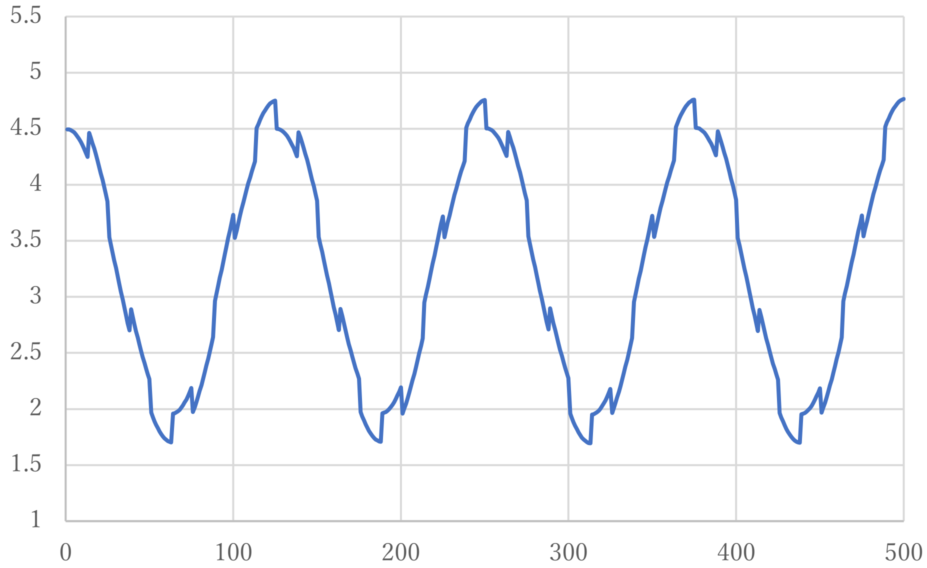

Fig.12 s_1 Estimation by Whitening

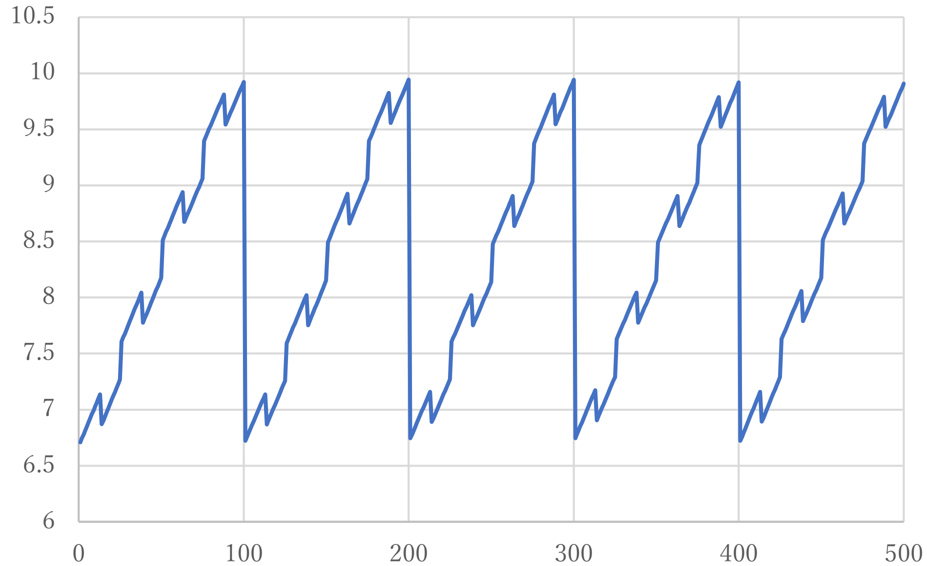

Fig.13 s_2 Estimation by Whitening

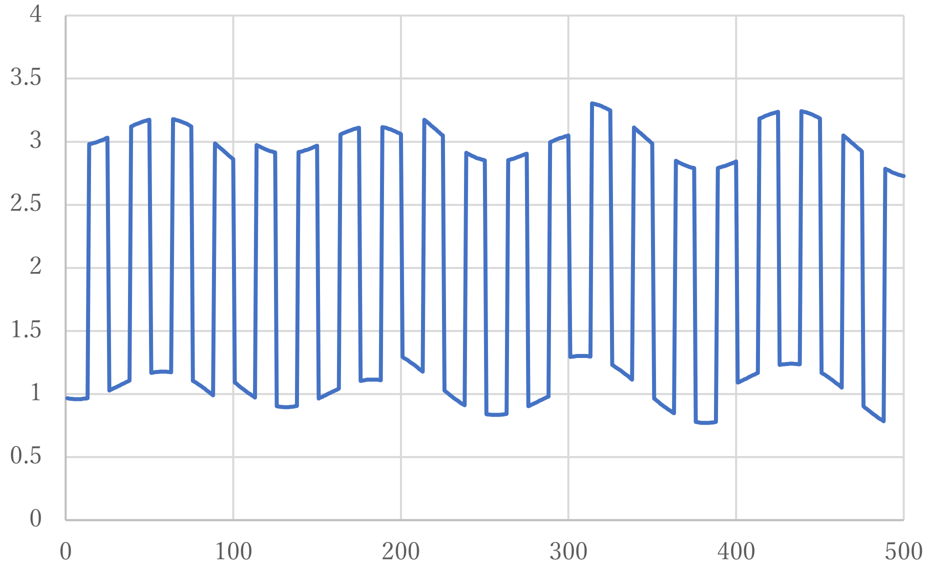

Fig.14 s_3 Estimation by Whitening

It can be estimated similar to the source signal shape too, if even not completely.

Conclusion

Although there is an indication for the insufficient separation, some extent effect is recognized. As the author, it is often useful for the feature extraction, sufficiently. The principal component analysis and whitening are based on the correlation, decorrelation. As it does not depend on the independent, it has correlations according to the period for the periodic waveforms, the author thinks that it comes from this.

For this simulation data, the effectivity by applying ICA(fastICA for maximum likelihood estimation[1]) has not been recognized more than this paper. Each waveform data is the sub-gaussian distribution (almost uniform distribution), so this point has no problems. As the self programming, the author has ever implemented by 2 super-gaussian distributions(calculated by the index distribution which is constructed from the uniform random numbers by using the inverse function method). According to the initial values, it was certain that the independent components rotate well, the bugs' possibility is hard to think. Up to now, the author has lost the interest more than this.

As a supplement, the spectrum(frequency transform) might be suitable rather than the whitening for this paper's simulation data. If there were assumptions that the signals has the periodicity and their periods were different from each other, it was not difficult to estimate A. As facing to the problem's solution, the method will not be limited by only one way. Actually, For the image processing patents which the author got, most of them are only combinations or applications of the methods.

In addition, refer the reference[2] about the principal component analysis. The author had special thanks to the reference. As this is a Japanese book, it is explained to understand very easily no more than that.

References

[1] Aapo Hyvärinen, Juha Karuhnen, Erkki Oja
Detailed Explanation Independent Component Analysis
Tokyo Denki Univ. Publishing Office. 2005.

[2] Yoshiyuki Wakui, Sadami Wakui
Illustrated to understand Multivariate Analysis
Nippon Jitsugyo Publishing Co. 2001.