Least Squares Solution

Introduction

The author considers 3 greatest tools for the engineering as follows.

* Least Squares Method
* Newton's Method
* Fourier Transform

This paper describes how to use least squares method and its general format.

Least Squares Method

The detail example of least squares method is formulas (1)～(3) which are for calculating the coefficients of linear function.

$$y = ax + b \quad (1)$$

$$a = \frac{\sum_{n=1}^{k} x_n y_n - \sum_{n=1}^{k} x_n \sum_{n=1}^{k} y_n}{k \sum_{n=1}^{k} {x_n}^2 - \left(\sum_{n=1}^{k} x_n\right)^2} \quad (2)$$

$$b = \frac{\sum_{n=1}^{k} {x_n}^2 \sum_{n=1}^{k} y_n - \sum_{n=1}^{k} x_n \sum_{n=1}^{k} x_n y_n}{k \sum_{n=1}^{k} {x_n}^2 - \left(\sum_{n=1}^{k} x_n\right)^2} \quad (3)$$

k ：data number

Generally, least squares method has an image like this analytical solution. However, real –least squares method used in the engineering is generalized curve fitting by an arbitrary curved surface. As it is, this paper says least squares solution for the curve fitting, hereafter.

Least Squares Solution

It is important for least squares method which can handle a curved surface (polynomial) with following extension.

* multivariable
* variable with arbitrary degree
* variable's combination at polynomial's term (multiplication, or division and

mapping of an arbitrary function)

The example (4) is that each term consists of multiplications. The author considers this as the general model.

$$f(x,y,z) = a_1x^4 + a_2x^2 + a_3y^3 + a_4y^2 + a_5y + a_6z + a_7xz + a_8yz + a_9 \quad (4)$$

The curved surface must be specified by calculating 9 coefficients for this example. 9 data set are necessary to solve the system of equations (5)

$$AX = B \quad (5)$$

$$A = \begin{bmatrix} {x_1}^4 & {x_1}^2 & {y_1}^3 & {y_1}^2 & y_1 & z_1 & x_1z_1 & y_1z_1 & 1 \\ {x_2}^4 & {x_2}^2 & {y_2}^3 & {y_2}^2 & y_2 & z_2 & x_2z_2 & y_2z_2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ {x_k}^4 & {x_k}^2 & {y_k}^3 & {y_k}^2 & y_k & z_k & x_kz_k & y_kz_k & 1 \end{bmatrix} \quad (6)$$

$$X = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_k \end{bmatrix} \quad (7)$$

$$B = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_k \end{bmatrix} \quad (8)$$

measurement data set：$(x_1, y_1, z_1, f_1), (x_2, y_2, z_2, f_2), \cdots (x_k, y_k, z_k, f_k)$

$$k = 9$$

The measurement data need only 9 set, at least, but actually, the data more than them should be got. This leads to least squares solution (9) that A is the excess factor's case.

$$A^\top AX = A^\top B \quad (9)$$

It is not this paper's drift for the proof that (9) minimizes the delta between the measurement data and the fitting curve (error sum of squares), so it will be omitted. In addition, $(A^\top A)^{-1}A^\top$ is sometimes called a pseudo inverse matrix. About the method for the system of equations' solution after this operation, Gaussian elimination is general for the numerical calculation.

If (5)'s format was applied, it did not matter that the union(addition)'s target is the transcendental function. (10) is an example.

$$f(x,y) = a_1x + a_2y + a_3e^{-x} \quad (10)$$

In another, there is a case to handle even if non-linear(refer to "Least Squares Solution's Example").

In summary, if the function format(model) was given, the least squares solution was able to be applied.

As a result, for the engineering, the modeling which can apply (9)'s format is important. This point seems that any technical book does not describe, then it becomes this paper's drift.

From the view of signal processing, the polynomial like (4) will be able to apply the fitting to whatever the curved surface is. For example, if there was a monochrome image taken by the camera, it was not impossible to apply the curve fitting to this which considered as the curved surface. However, if it was realized, Fourier transform was used, generally. The polynomial approximation has a common experience that Fourier series can express the arbitrary curve.

＊Polynomial: variable's degree ⇔ Fourier series' harmonic degree

i.e. extreme value's number ≒ frequency

Once again, improve the modeling to use the tool, it is considered as the way for the engineering. In addition, there is a view that the curve fitting is an information compression. This comes from a reason that the measurement data can express the curved face expressed by a few coefficients. From the insight above, it may lead to various applications.